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Fixed order controller design via linear programming

King-Chi Kelly Yip
Iowa State University

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King-Chi Kelly Yip

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King-Chi Kelly Yip

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ABSTRACT

Fixed order controllers, such as PD, PI and PID controllers, are the most commonly used in hardware implementations. Therefore, control engineers are often given the task to design fixed order controllers with time domain specifications as the system design requirements. To solve this problem, a systematic fixed order controller design process employing mathematical programming is developed in this thesis. The design process is based on fixed order pole assignment problem. Natural frequencies and minimum damping ratio are the design parameters of the developed controller design process. The design parameters are insightful since damping ratio and frequencies are closely related to time domain specifications. The interval polynomial search algorithm is formulated to maximize the interval characteristic polynomial within the design domain defined by the design parameters on the s-plane. The Edge Theorem is applied to ensure the performance and stability of the interval characteristic polynomial [3]. It is followed by solving the optimal controller, using the linear or non-linear programming with linear constraints technique. This proposed method ensures that the controller will attain the acceptable performance and stability even if the plant dynamic consists of some level of uncertainties. The advantage of this method is the freedom of the choices of design emphasis.

CHAPTER 1: INTRODUCTION

Fixed order controllers, such as Proportional-Integral-Derivative (PID) controllers, are the most used in today's systems. However, the techniques of designing an optimal or robust fixed order controller are not often studied. Keel and Bhattacharyya [1, 2] had formulated the fixed order controller design technique using linear programming approach in their recent research work. The foundation of their formulation was based on the fixed order pole assignment (FOPA) problem and the fixed order model matching (FOMM) problem, which started with an exact target closed loop characteristic equation and then relaxed the constraints to a target interval of characteristic equation or transfer function. The Edge Theorem [3] is used to ensure the performance and stability of the interval characteristic equation. The relaxation from an exact target closed loop characteristic equation to a target interval of characteristic equation transforms the controller design problem to a constrained linear programming (LP) problem. The details of Keel and Bhattacharyya's formulation will be explained in Chapter 4.

However, Keel and Bhattacharyya's formulation can be implemented only if the closed loop target interval characteristic equation is known. Control engineers are often given the task to design controllers that satisfy performance requirements in time response or frequency response. The transformation from transfer function or characteristic equation to time response is simple. However, if the time response is specified in a range, its mapping to the characteristic equation is not explicit, making Keel and Bhattacharyya's LP approach to controller design impractical.

The objective of this project is to formulate the fixed order controller design in to a systematic and insightful process. The input parameters of the process are bounds of damping ratio and natural frequency of the roots of the characteristic polynomial. A target interval characteristic polynomial search method is introduced to close the gap between the time domain design specifications and the mathematical LP approach. This target characteristic polynomial search method is named as the "Interval Polynomial Search Algorithm" in this thesis. After the interval polynomial is obtained, the controller is solved using the LP approach to fixed order controller design proposed by Keel and Bhattacharyya. This controller design process is more insightful, since the design parameters are well correlated with system response. Meanwhile, all the advantages of linear programming approach to controller design are maintained.

In this thesis, the fundamental theories, such as the FOPA, FOMM and the Edge Theorem [3] will be first introduced. The developed algorithm to search for the target interval characteristic polynomial, the Interval Polynomial Search Algorithm, will be described in Chapter 3. This interval polynomial search algorithm is the main contribution of this project. The fixed order controller design process incorporating the LP approach by Keel and Bhattacharyya and the interval polynomial search algorithm will be described in Chapter 4. The formulation of the process is based on single input single output (SISO), linear time invariant (LTI) system. A step-by-step illustration of the controller design process for a second order system is shown in the chapter as well. The results of the example will be compared with a linear quadratic regulator (LQR) design in Chapter 5. In addition, the similarities and differences of the design process and LQR will be discussed. To demonstrate that the new approach is applicable to higher order system, a 4^{th} order aircraft longitudinal mode problem will be demonstrated in Chapter 6.

CHAPTER 2: BACKGROUND THEORIES

The fixed order controller design process developed in this thesis, as well as the linear programming approach to fixed order control design formulated by Keel and Bhattacharyya, are built on the foundation of the fixed order model matching (FOMM) problem, the fixed order pole assignment (FOPA) problem, linear programming (LP) and the Edge Theorem [3]. In this chapter, the background of the above theories will be explained.

2.1 Fixed Order Model Matching (FOMM)

Consider a single input single output (SISO) linear time-invariant (LTI) system as shown in Figure 1 below, where *C(s)* represents an m^{th} order fixed order controller, and *P(s)* represents an n^{th} order plant. The transfer functions of *C(s)* and *P(s)* are shown in Equation (2.1) and (2.2) respectively.

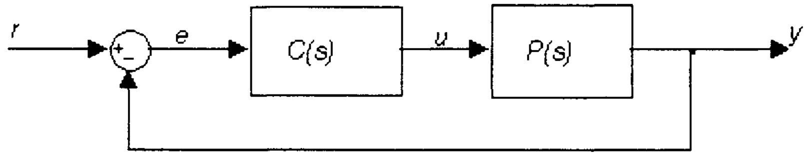

Figure 1. SISO feedback system.

$$C(s)=\frac{n_c(s)}{d_c(s)}=\frac{a_m s^m + a_{m-1}s^{m-1}+\ldots+a_0}{b_m s^m + b_{m-1}s^{m-1}+\ldots+b_0} \tag{2.1}$$

$$P(s)=\frac{n(s)}{d(s)}=\frac{n_n s^n + n_{n-1}s^{n-1}+\ldots+n_0}{d_n s^n + d_{n-1}s^{n-1}+\ldots+d_0} \tag{2.2}$$

A fixed order model matching problem involves picking a fixed order controller *C(s)* such that the coefficients of the resulting transfer function's numerator and denominator match exactly to the target system's coefficients. For the controller, *C(s)*, and the plant dynamics, *P(s)*, defined in Equation (2.1) and (2.2), the resulting closed loop transfer function *G(s)* can be defined as Equation (2.3). If the model of the target system is defined as Equation (2.4), the coefficients of the controller can be solved from Equation set (2.5).

$$\begin{aligned} G(s) &= \frac{C(s)P(s)}{1+C(s)P(s)} \\ &= \frac{n_c(s)n(s)}{n_c(s)n(s)+d_c(s)d(s)} \\ &= \frac{n_n a_m s^{n+m} + (n_n a_{m-1} + n_{n-1}a_m)s^{n+m-1}+\ldots+n_0 a_0}{(n_n a_m + d_n b_m)s^{n+m} + \begin{pmatrix} n_n a_{m-1} + n_{n-1}a_m \\ + d_n b_{m-1} + d_{n-1}b_m \end{pmatrix} s^{n+m-1}+\ldots+(n_0 a_0 + d_0 b_0)} \end{aligned} \tag{2.3}$$

$$G_T(s)=\frac{\phi_{T\_n+m}s^{n+m}+\phi_{T\_n+m-1}s^{n+m-1}+\ldots+\phi_{T\_1}s+\phi_{T\_0}}{\delta_{T\_n+m}s^{n+m}+\delta_{T\_n+m-1}s^{n+m-1}+\ldots+\delta_{T\_1}s+\delta_{T\_0}} \tag{2.4}$$

$$
\begin{aligned}
\phi_{T\_n+m} &= n_n a_{m\,m} \\
\phi_{T\_n+m-1} &= n_n a_{m-1} + n_{n-1} a_m \\
&\vdots \\
&\vdots \\
&\vdots \\
\phi_{T\_0} &= n_0 a_0 \\
\delta_{T\_n+m} &= n_n a_m + d_n b_m \\
\delta_{T\_n+m-1} &= n_n a_{m-1} + n_{n-1} a_m + d_n b_{m-1} + d_{n-1} b_m \\
&\vdots \\
&\vdots \\
&\vdots \\
\delta_{T\_0} &= n_0 a_0 + d_0 b_0
\end{aligned}
\tag{2.5}
$$

Equation set (2.5) consists of $2(m+n+1)$ equations and $2(m+1)$ unknowns. This implies that the equation set is either inconsistent or redundant. For a unique solution to the above equation set (Equation (2.5)), only $2(m+1)$ linearly independent equations are needed. Therefore, if there are solutions to the equation set, all of the $2(m+n+1)$ equations must intersect at the same point. This implies that the equation set is over determined. This is the case of redundancy. If there are some equations parallel to some other equations in the set, or if the equations do not intersect at the same point, there is no solution to the equation set. This is the case of inconsistency.

2.2 Fixed Order Pole Assignment (FOPA)

Pole assignment, also known as pole placement, involves placing the closed loop poles to the specific desired locations, and hence achieving desired characteristics. The fixed order pole assignment problem is a sub-problem of the fixed order model matching (FOMM) problem. By choosing not to assign zeros, the problem becomes FOPA. Consider the same single input single output (SISO) linear time-invariant (LTI) system as shown in Figure 1, where the transfer functions of *C(s)* and *P(s)* are defined in Equation (2.1) and (2.2) respectively. The resulting closed loop characteristic equation of Figure 1 is stated in Equation (2.6).

$$\begin{aligned}\delta(s) &= n(s)n_c(s) + d(s)d_c(s) \\ &= (n_n a_m + d_n b_m)s^{n+m} \\ &+ (n_n a_{m-1} + n_{n-1}a_m + d_n b_{m-1} + d_{n-1}b_m)s^{n+m-1} \\ &+ \ldots + (n_0 a_0 + d_0 b_0)\end{aligned} \tag{2.6}$$

Suppose the desired pole locations of the resulting system are the roots of the closed loop characteristic equation, $\delta_T(s)$, which is stated in Equation (2.7) and the dynamics of the plant are known, i.e. *n's* and *d's* are known. Furthermore, the plant has a higher order than the controller, i.e. $m \leq n-1$, the controller coefficients, *a's* and *b's*, can be chosen by solving the set of linear equations in Equation (2.8).

$$\delta_T(s) = \delta_{T\_n+m}s^{n+m} + \delta_{T\_n+m-1}s^{n+m-1} + \ldots + \delta_{T\_1}s + \delta_{T\_0} \tag{2.7}$$

$$\begin{aligned}\delta_{T\_n+m} &= n_n a_m + d_n b_m \\ \delta_{T\_n+m-1} &= n_n a_{m-1} + n_{n-1}a_m + d_n b_{m-1} + d_{n-1}b_m \\ &\vdots \\ &\vdots \\ &\vdots \\ \delta_{T\_0} &= n_0 a_0 + d_0 b_0\end{aligned} \tag{2.8}$$

As it was mentioned before, the fixed order pole assignment (FOPA) problem is a subset the fixed order model matching (FOMM) problem. The FOMM can be simplified and solved as a FOPA. In fact, by comparing Equation set (2.3) and (2.6), it can be identified that the denominator of the target transfer function of FOMM is identical to the close loop characteristic equation of FOPA in Equation (2.6). Therefore, if the coefficients of the numerator of $G_T(s)$ are set up in the way that is consistent to the coefficients of the denominator of $G_T(s)$, i.e. all the equations in Equation (2.5) intersect at the same point; the FOMM problem can be solved using Equation set (2.8).

2.3 Linear Programming

Linear programming is a method to optimize, either minimizing or maximizing a linear function while fulfilling a set of linear equality or inequality constraints. In general, the linear programming problem can be presented as Equation (2.9), where x is a vector of decision variables to be solved for, A, and b are matrix and vector of constants respectively, and $f(x)$ is a linear function of x.

$$\begin{gathered} \underset{x}{Min}\ f(x) \\ subject\ to\ Ax \geq b \end{gathered} \tag{2.9}$$

2.4 Edge Theorem

The Edge Theorem was founded by Bartlett, Hollot and Lin. The theorem relates a family of an n^{th} degree polynomial with the root locations in the s-plane. Its results are useful in system performance and stability analysis [3]. Consider an n^{th} order LTI system with some of its parameters vary within a range. The system consists of a whole family of n^{th}-degree characteristic polynomial, and each member of the polynomial family has the format represented in Equation (2.10).

$$f(s) = s^n + c_1 s^{n-1} + \cdots + c_n \tag{2.10}$$

s in Equation (2.10) represents a Laplace variable, which is a complex number, while $c's$ in the equation are real numbers. The associating vector of coefficients of the polynomial in Equation (2.10) is:

$$f \equiv \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \tag{2.11}$$

$\Omega \subset R^n$ is defined to be the set of associating vectors that correspond to the entire family of polynomials. The set, Ω, is the representative of the polynomial family in coefficient space [3].

In [3], Ω is assumed to be an m-dimensional polytope; i.e., the convex hull of a finite number of vertices. The "exposed sets" of Ω are those set of vectors of coefficients composed $\Omega \subset H$, where H is a nontrivial supporting hyperplane [3]. The one dimensional exposed sets are called "exposed edges", while the two dimensional exposed sets are called the "exposed faces". The "root space" which is the root locations contained in subsets of Ω is defined as:

Definition:

Consider any $W \subset \Omega$, the subset $R(W) \subset \mathbb{C}$, $R(W) = \{s : f(s) = 0, f \in W\}$ is the root space [3].

Consider an arbitrary $f \in \Omega$, let s_r and s_c represent the sets of real roots and complex roots of *f* respectively.

Lemma

If $s_r \in R(\Omega)$ is real, then there exists an exposed edge $E \subset \Omega$ such that $s_r \in R(E)$. Similarly, if $s_c \in R(\Omega)$ is complex, then there exists an exposed face $F \subset \Omega$ such that $s_c \in R(F)$ [3].

To conclude, Bartlett, Hollot and Lin state the Edge Theorem as:

Theorem

Consider $D \subset \mathbb{C}$ to be a simply connected domain. $R(\Omega)$ is contained in *D* if and only if all the exposed edges and exposed faces of Ω are contained in *D* [3].

An example on how to apply the Edge Theorem taken from [3] is as follows:

Example

Suppose $\Omega \subset R^3$ represents a polytope of 3rd order polynomials and it is a convex hull of four vertex polynomials. *D* is defined as the open left-half of the complex plane. The four vertex polynomials are stated in Equation (2.12).

$$\begin{aligned} P_1(s) &= s^3 + 9.77s^2 + 30.6s + 18.27 \\ P_2(s) &= s^3 + 15s^2 + 75s + 125 \\ P_3(s) &= s^3 + 8.96s^2 + 21.91s + 15.61 \\ P_4(s) &= s^3 + 11.43s^2 + 20.2s + 82.5 \end{aligned} \tag{2.12}$$

The exposed edges and exposed faces of the root space, $R(\Omega)$, are the six straight lines connecting the four exposed vertex polynomials in Equation (2.12). The root space of the line segment connecting vertex polynomial $P_i(s)$, and vertex polynomial $P_j(s)$, is found by solving for the roots of $(1-\lambda)P_i(s) + \lambda P_i(s)$) as λ is swept from zero to one . The root space, $R(\Omega)$, can be obtained by solving for the root space of all of the six line segments. For this example, the roots of Equation set (2.13) as λ is swept from zero to one form the root space, $R(\Omega)$. Figure 2 shows the root space created. The root space of all the exposed edges is contained in *D*. Therefore the root space of the polytope is also contained in *D*.

$$\begin{aligned} &(1-\lambda)P_1(s) + \lambda P_2(s) = 0 \\ &(1-\lambda)P_1(s) + \lambda P_3(s) = 0 \\ &(1-\lambda)P_1(s) + \lambda P_4(s) = 0 \\ &(1-\lambda)P_2(s) + \lambda P_3(s) = 0 \\ &(1-\lambda)P_2(s) + \lambda P_4(s) = 0 \\ &(1-\lambda)P_3(s) + \lambda P_4(s) = 0 \quad \text{, where } 0 \le \lambda \le 1 \end{aligned} \tag{2.13}$$

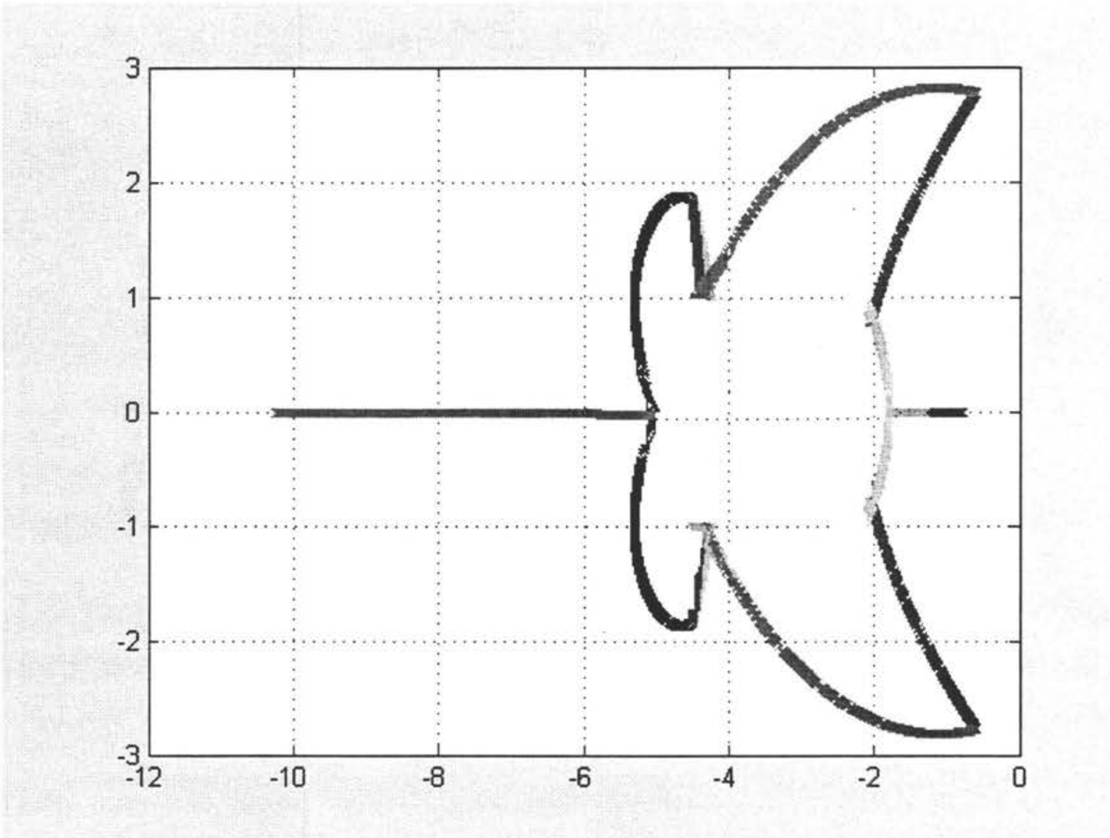

Figure 2. Root space created from the Edge Theorem example.

CHAPTER 3: THE INTERVAL POLYNOMIAL SEARCH ALGORITHM

The interval polynomial search algorithm described in this chapter searches for the interval polynomial with a minimum damping ratio, a minimum natural frequency and a maximum frequency as the inputs. This algorithm is the bridge that connects the gap between the time domain design specifications and the mathematical linear programming approach to fixed order controller design by Keel and Bhattacharyya [1, 2].

3.1 The Formulation

The formulation of the algorithm is taking advantage of the Edge Theorem [3] and the convexity nature of the polytope formed from the vertex poles. Suppose an m^{th} order controller is selected. The resulting system is in the $(m+n)^{th}$ order. The closed loop characteristic equation is:

$$\delta(s) = \delta_{n+m} s^{n+m} + \delta_{n+m-1} s^{n+m-1} + \ldots + \delta_1 s + \delta_0 \tag{3.1}$$

The above equation is normalized and becomes:

$$\delta(s) = s^{n+m} + \alpha_{n+m-1} s^{n+m-1} + \ldots + \alpha_1 s + \alpha_0 \tag{3.2}$$

Equation (3.2) has *(n+m)* coefficients. The normalized interval polynomial is stated in Equation (3.3), where $\overline{\alpha}$ and $\underline{\alpha}$ represent the lower and upper bounds of the polynomial coefficients.

$$\delta(s) = s^{n+m} + \left[\underline{\alpha}_{n+m-1} \quad \overline{\alpha}_{n+m-1}\right]s^{n+m-1} + \ldots + \left[\underline{\alpha}_1 \quad \overline{\alpha}_1\right]s + \left[\underline{\alpha}_0 \quad \overline{\alpha}_0\right] \tag{3.3}$$

For a *(n+m)*th order interval polynomial, there are $2^{(m+n)}$ vertex polynomials, which are made up by the combinations of all the lower and upper bounds of each characteristic equation coefficient. Each of these polynomials has *(m+n)* roots. In order to have an interval polynomial that assures the time domain specifications, each of the roots must satisfy the defined range for damping ratio and natural frequencies. These form the constraints of the nonlinear programming problem.

For the widest selection of controller to choose from in the next stage, the objective of this nonlinear programming problem is to obtain the largest possible range for each characteristic equation coefficient. The formulation of the problem is stated in Equation (3.4), where V_j defines as the j^{th} vertex polynomial, $damping\left[V_j\right]$ and $frequency\left[V_j\right]$ define as the damping ratios and the natural frequencies of the roots of V_j respectively.

$$\begin{aligned} &\underset{\underline{\alpha}.\alpha}{Max} \prod_{i=0}^{n+m-1} \left(\overline{\alpha}_i - \underline{\alpha}_i\right) \\ &subject\ to \quad damping\left[V_j\right] \geq \zeta_{\min} \\ &\quad \omega_{n\_\max} \geq frequency\left[V_j\right] \geq \omega_{n\_\min} \qquad ; for\ \forall j \end{aligned} \tag{3.4}$$

3.1 The Solution

The nonlinear programming problem (Equation (3.4)) is solved by the barrier method to ensure that the polytope of the interval polynomial lies entirely within the defined design domain [13]. An initial guess of the bounds interval polynomial coefficient, $\underline{\alpha}$ and $\overline{\alpha}$, is needed as a starting point to solve for the nonlinear problem in Equation (3.4). The guess must satisfy all the inequality constraints in the problem because the barrier method can only solve the optimization problem with a feasible initial starting point [13]. For an n^{th} order polynomial, where $n > 2$, the solutions obtained for Equation (3.4) are dependent on the starting values. It is because the constraints, which define the feasible domain in the coefficient space, are highly nonlinear for high order polynomial. The detailed procedures to solve the nonlinear programming problem in Equation (3.4) are discussed in [13].

CHAPTER 4: THE LINEAR PROGRAMMING APPROACH TO FIXED ORDER CONTROLLER DESIGN

The advantages of the linear programming approach to fixed order design include: single or multiple design aspects may be optimized while the performance design requirements can be fulfilled and the computationally feasibility [1, 2]. Keel and Bhattacharyya [1, 2] had developed the linear programming approach to fixed order design based on the FOPA and FOMM problem. However, their approach did not address how to obtain the constraint values, i.e. the family of target characteristic polynomials. In this chapter, Keel and Bhattacharyya's formulation will be elaborated. A fixed order controller design process, which incorporates Keel and Bhattacharyya's formulation and the interval polynomial search algorithm in Chapter 3, will be proposed. This fixed order controller design process involves choosing 3 design parameters: minimum damping ratio, minimum natural frequency, and maximum natural frequency. These parameters are then converted into an interval polynomial using the interval polynomial search algorithm described in Chapter 4. The resulting interval polynomial is the constraints needed in the linear programming approach to fixed order design. The proposed controller design process is closely connected to the time-domain performance requirements because the designing parameters are damping ratio and natural frequencies, and their correlations to time-domain responses are well understood. This design process will be illustrated using a 2^{nd} order system.

4.1 The Background References

Keel and Bhattacharyya formulated a linear programming approach to fixed order controller design based on the FOPA and FOMM problems. It starts with a single target characteristic polynomial. However, for an m^{th} order controller with an n^{th} order plant ($m \leq n-1$), it is generally difficult to attain the exact target characteristic polynomial. To solve this problem, each of the characteristic equation coefficients is expanded to an interval of values by relaxing the requirements. From this procedure, an interval polynomial family is obtained. A polynomial family is described in Equation (4.1). This interval polynomial family forms a set of vertex polynomials, which are all possible combinations of lower and upper bound of each polynomial coefficient. According to the Edge Theorem [3], as described in chapter 2.4, these vertex polynomials form a root space. As long as all of the coefficients lie within their ranges, the roots of the characteristic equation lie within the root space [3].

$$\Delta_T = \left\{ \begin{array}{c} \delta_T(s) = \delta^T_{n+m} s^{n+m} + \delta^T_{n+m-1} s^{n+m-1} + \ldots + \delta^T_0 \\ \delta^{T-}_i \leq \delta^T_i \leq \delta^{T+}_i,\ for\ all\ i \end{array} \right\} \tag{4.1}$$

For a closed-loop characteristic equation (Equation (2.6)), the constraints of the controller become Equation set (4.2). Equation (4.3) shows the corresponding matrix presentation of the constraint set.

$$
\begin{aligned}
\delta^{T-}_{n+m} &\le n_n a_m + d_n b_m \le \delta^{T+}_{n+m} \\
\delta^{T-}_{n+m-1} &\le n_n a_{m-1} + n_{n-1} a_m + d_n b_{m-1} + d_{n-1} b_m \le \delta^{T+}_{n+m-1} \\
&\vdots \\
&\vdots \\
&\vdots \\
\delta^{T-}_{0} &\le n_0 a_0 + d_0 b_0 \le \delta^{T-}_{0}
\end{aligned}
\tag{4.2}
$$

$$
\underbrace{\begin{bmatrix} \delta^{T-}_{n+m} \\ \delta^{T-}_{n+m-1} \\ \delta^{T-}_{n+m-2} \\ \vdots \\ \vdots \\ \vdots \\ \delta^{T-}_{1} \\ \delta^{T-}_{0} \end{bmatrix}}_{\delta_{\_l}}
\le
\underbrace{\begin{bmatrix}
n_n & & & & d_n & & & \\
n_{n-1} & n_n & & & d_{n-1} & d_n & & \\
\vdots & n_{n-1} & \ddots & & \vdots & d_{n-1} & \ddots & \\
\vdots & \vdots & & n_n & \vdots & \vdots & & d_n \\
n_0 & \vdots & & n_{n-1} & d_0 & \vdots & & d_{n-1} \\
 & n_0 & & \vdots & & d_0 & & \vdots \\
 & & \ddots & \vdots & & & \ddots & \vdots \\
 & & & n_0 & & & & d_0
\end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} a_m \\ a_{m-1} \\ \\ a_0 \\ b_m \\ b_{m-1} \\ \\ b_0 \end{bmatrix}}_{K}
\le
\underbrace{\begin{bmatrix} \delta^{T+}_{n+m} \\ \delta^{T+}_{n+m-1} \\ \delta^{T+}_{n+m-2} \\ \vdots \\ \vdots \\ \vdots \\ \delta^{T+}_{1} \\ \delta^{T+}_{0} \end{bmatrix}}_{\delta_{\_u}}
$$

$$
\delta_{\_l} \le AK \le \delta_{\_u} \tag{4.3}
$$

The controller design problem is transformed into a linear programming problem with linear constraints. An example of a linear programming problem is minimizing the sum of the gain values. The format of the problem is shown in Equation (4.4).

$$
\begin{aligned}
&\underset{a,b}{Min}\ f(K) \\
&subject\ to\ AK \le \delta_{\_u} \\
&\qquad\qquad AK \ge \delta_{\_l}
\end{aligned}
\tag{4.4}
$$

4.2 The Fixed Order Controller Design Process

The LP approach proposed by Keel and Bhattacharyya is computationally simple. However, there is a gap between the physical design requirement and the approach. It is not trivial to obtain the interval characteristic equation needed in the LP approach for some given bounds of time response specification. The main criterion to apply the linear programming approach to controller design is the known interval polynomial. The design process developed in this chapter makes a closer connection between the design process and the time-domain design specifications. Instead of arbitrarily choosing desired pole locations or intervals of the desired characteristic equation coefficients, one needs only to choose the minimum damping ratio and a range of natural frequencies of the poles of the resulting system. Since there is a close relationship between damping ratio and natural frequency with time-domain performances, this design process should be more insightful.

The fixed order controller design process is summarized as follows:

1. Choose ranges of damping and frequency that satisfies the performance requirements
2. Assume the order of the controller
3. Obtain the target interval characteristic polynomial with the interval polynomial search algorithm described in Chapter 3.
4. Apply the linear programming approach to fixed order controller design as proposed by Keel and Bhattacharyya.

Steps 1 and 2 will be explained in detail in this chapter. Step 3 is the transition from design parameters to the linear programming approach to fixed order controller design. It closes the gap between the well-understood system characteristics, such as damping ratio and natural frequency, and the mathematical approach to controller design.

4.2.1 Choosing Frequency and Damping

Frequently, control engineers are given the task to design controllers that satisfy a set of time response requirements. Requirements may be given as the maximum overshoot, M_p, maximum settling time, t_s, peak time, t_p or rise time, t_r. These physical response requirements are related with the damping ratio, ζ, and natural frequency, ω_n, of the system. A control engineer may choose the minimum damping ratio, ζ, and minimum natural frequency, ω_n that fulfill the minimal requirements to design the controller. The approximated relation of damping ratio and frequency to those physical responses are stated in Equation (4.5), (4.6), (4.7) and (4.8).

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \tag{4.5}$$

$$t_s = \frac{-\ln(0.02)}{\zeta\omega_n}; \text{ for 2 \% settling amplitude} \tag{4.6}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{4.7}$$

$$t_r = \frac{\ln(0.9)-\ln(0.1)}{\zeta\omega_n}; \text{ rise from 10\% to 90\%} \tag{4.8}$$

With the above relations stated in Equation (4.5)-(4.8), the time-domain specifications can be easily transformed to the s-plane or the complex plane. The illustration is shown in Figure 3. The unshaded area shows the domain, which satisfies the specifications

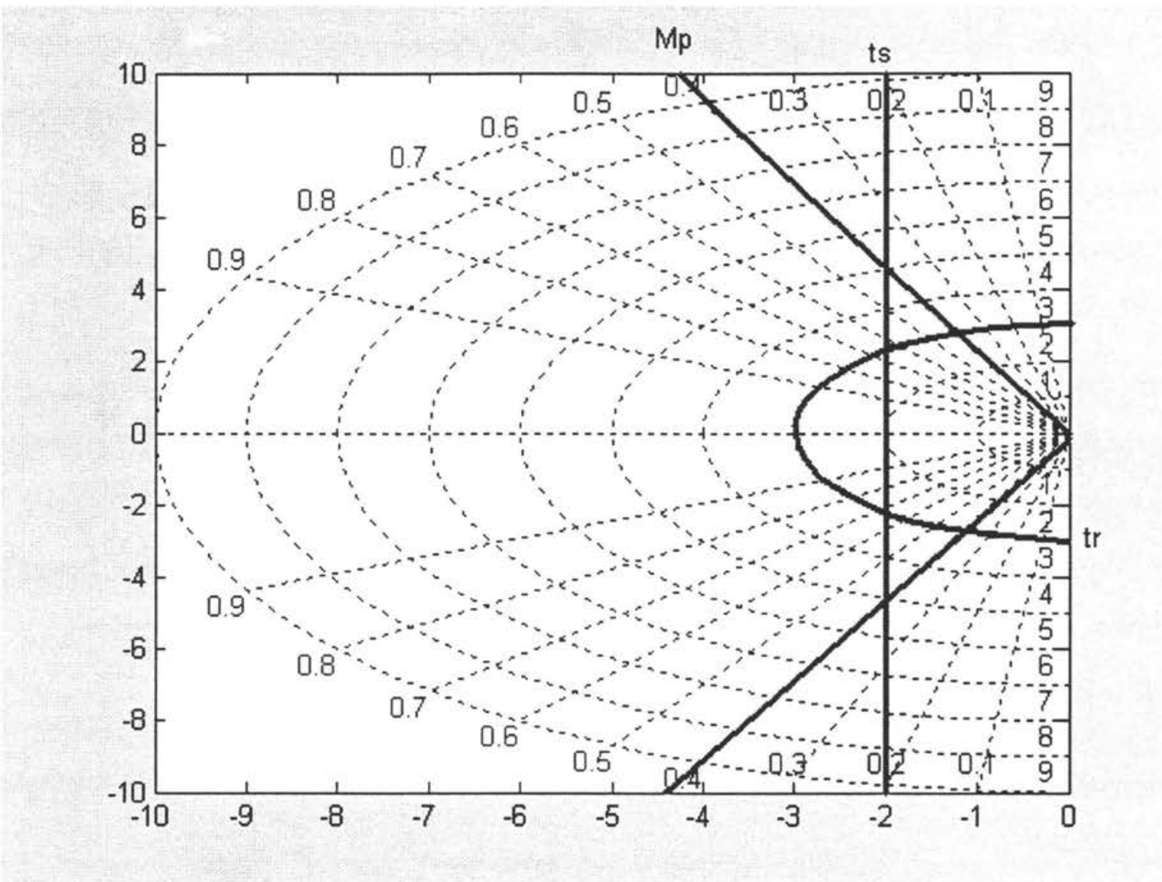

Figure 3. Map of time-domain specifications on s-plane.

However, apart from the minimum damping ratio and natural frequency, a maximum natural frequency may be defined as well for the controller design, because high frequency may result in high control effort.

4.2.2 Order of Controller

Recall that an m^{th} order controller, $C(s)$ has a format as:

$$C(s) = \frac{n_c(s)}{d_c(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0} \tag{4.9}$$

Therefore, a 0^{th} order controller is a pure gain controller. The resulting system is in the $(m+n)^{th}$ order. A designer may want to keep the order of the controller as low as possible. So, a lowest order controller may be used to start with. If the lowest order controller does not work, one can increase the order of the controller until desired performances are achieved.

4.3 Illustrative Example (2^{nd} Order System)

This example is taken from Etkin[5]. Consider the short period longitudinal mode of a Boeing 747, the detailed flight conditions and plane configurations are described in Appendix A. The time response design specifications are as follow:

- Overshoot, $M_p \leq 0.15$
- Settling time (1%), $t_s < 1.00 \sec$

The transfer function of angle of attack to elevator deflection is given as Equation (4.10).

$$\frac{\alpha}{\delta_e} = G(s) = \frac{0.03257\ s\ + 1.1052}{s^2 + 0.9186 s + 1.064} \frac{rad}{rad} \tag{4.10}$$

Open loop Plant Analysis

The pole-zero map of Equation (4.10) is plotted in Figure 4. The damping ratio and the natural frequency are 0.445 and 1.03 rad/s respectively. The step response and the bode plots of the open loop plant are plotted in Figure 5 and 6. It shows that the system does not meet any of the above specifications. The overshoot and settling time are about 21% and 8.5 sec respectively. A controller is needed to improve the performance.

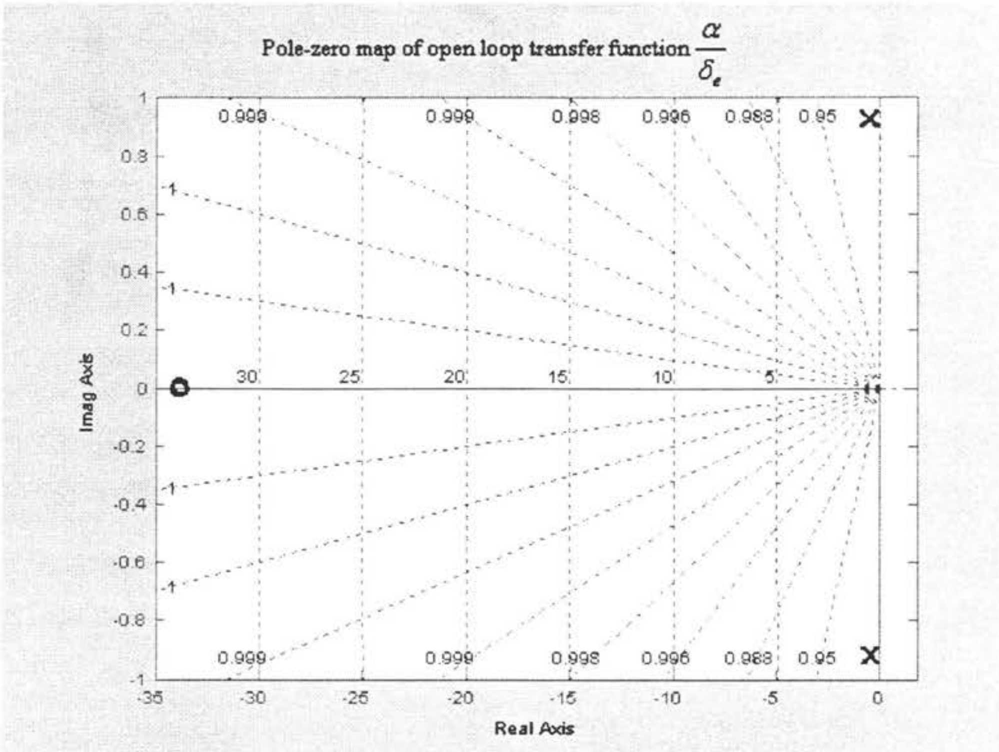

Figure 4. Pole-zero map of the 2nd order open loop plant.

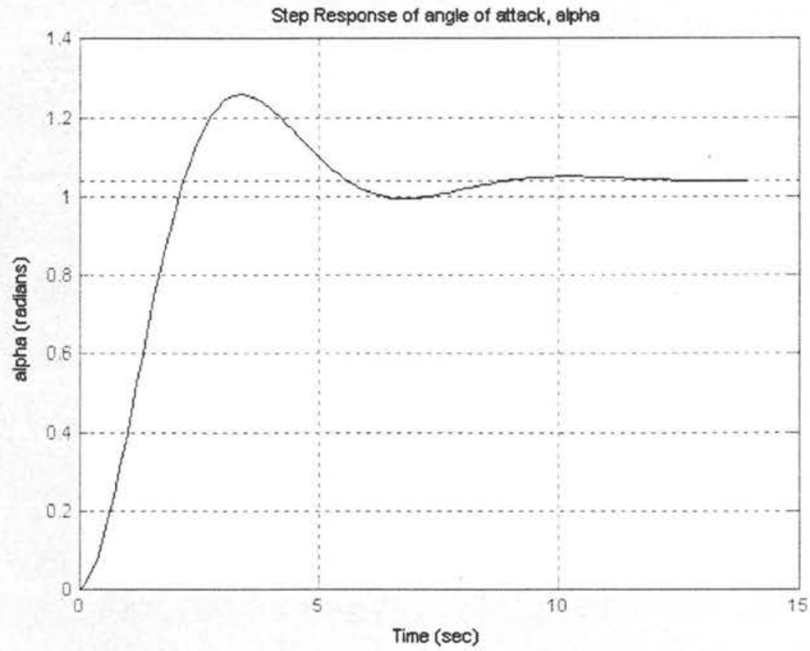

Figure 5: Step response of the 2^{nd} order open loop plant.

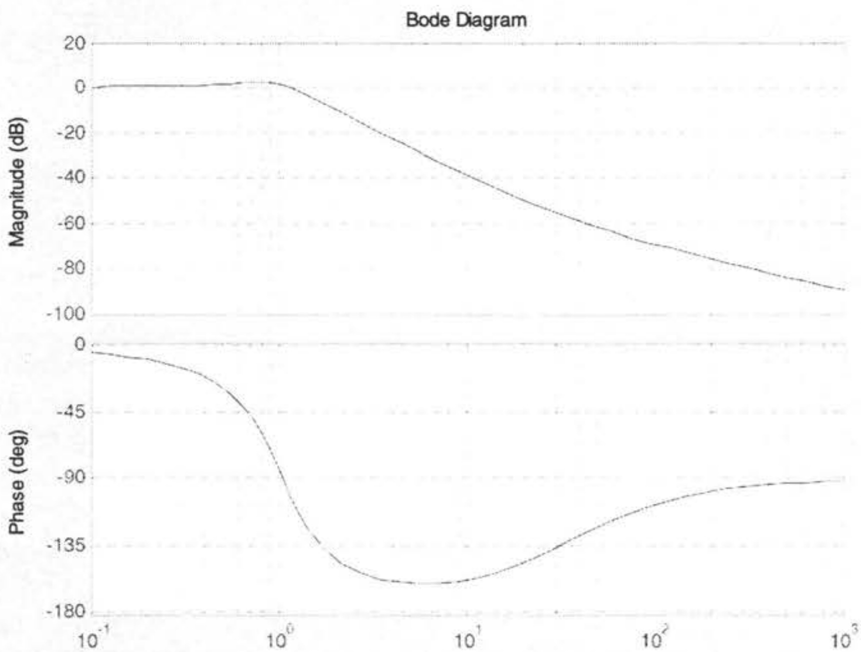

Figure 6: Bode plots of the 2^{nd} order open loop plant.

Step 1

Translating the above specifications into a minimum requirement of damping ratio and natural frequency gives: $\zeta \geq 0.5169$ and $\zeta\omega \geq 4.605$. So the minimum damping ratio chosen is $\zeta_{\min} = 0.52$ and the range for the natural frequency is $\omega_n = [4.75 \quad 15]$ rad/s. The minimum damping ratio and the lower bound of the natural frequency are chosen according to the minimum requirements solved above. 15 rad/s is arbitrary chosen to be the upper bounded to limit the control effort.

Step 2

Assume that a pure gain (Proportional) controller will satisfy the design specification, and thus $m=0$.

Step 3

Following the assumption made in Step 2, the order of the resulting system is 2. The target interval polynomial is:

$$\delta_T(s) = s^2 + [\underline{\alpha}_1 \quad \overline{\alpha}_1]s + [\underline{\alpha}_0 \quad \overline{\alpha}_0] \tag{4.11}$$

There are four vertex polynomials yielded from the target interval polynomial. They are:

$$V_1(s) = s^2 + \underline{\alpha}_1 s + \underline{\alpha}_0 \tag{4.12}$$

$$V_2(s) = s^2 + \underline{\alpha}_1 s + \overline{\alpha}_0 \tag{4.13}$$

$$V_3(s) = s^2 + \overline{\alpha}_1 s + \underline{\alpha}_0 \tag{4.14}$$

$$V_4(s) = s^2 + \overline{\alpha}_1 s + \overline{\alpha}_0 \tag{4.15}$$

Using the interval polynomial search algorithm described in Chapter 3, the formulation of the problem to find the range of each coefficient (or the target interval characteristic polynomial) is:

$$\begin{gathered}
\underset{\underline{\alpha}.\alpha}{Max}\ (\overline{\alpha}_1 - \underline{\alpha}_1)(\overline{\alpha}_0 - \underline{\alpha}_0) \\
subject\ to\quad damping[V_1(s)] \geq 0.52 \\
damping[V_2(s)] \geq 0.52 \\
damping[V_3(s)] \geq 0.52 \\
damping[V_4(s)] \geq 0.52 \\
15 \geq frequency[V_1(s)] \geq 4.75 \\
15 \geq frequency[V_2(s)] \geq 4.75 \\
15 \geq frequency[V_3(s)] \geq 4.75 \\
15 \geq frequency[V_4(s)] \geq 4.75
\end{gathered} \tag{4.16}$$

The constraint space of Equation (4.16) is illustrated in Figure 7. The outer bounds of the graph show the actual constraints on α_1 and α_0.

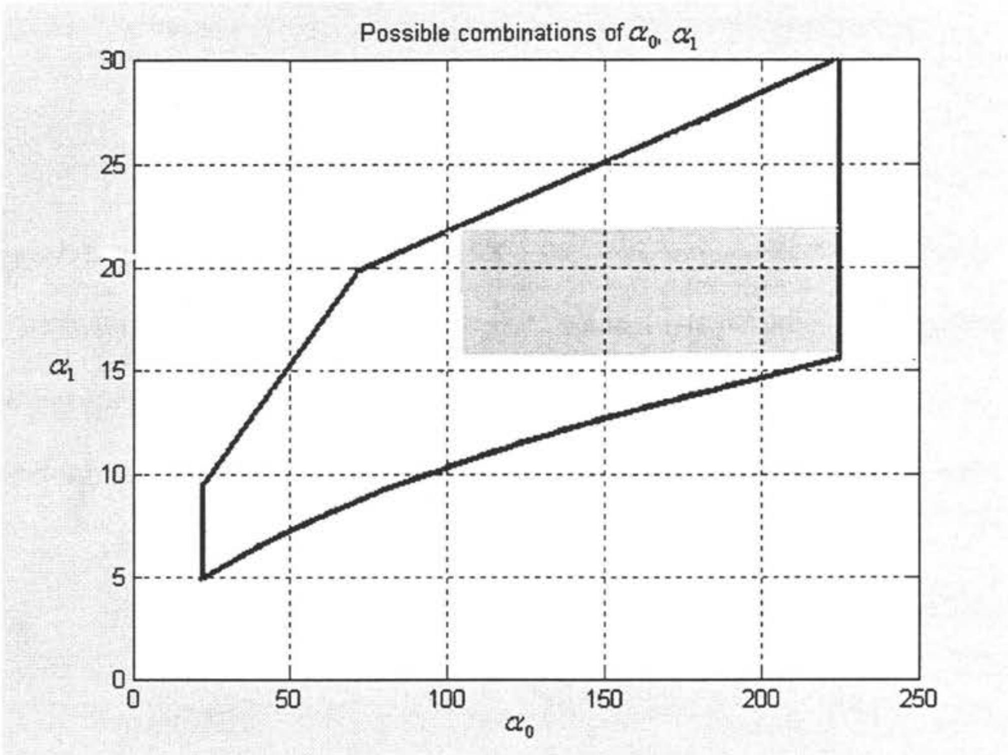

Figure 7. Plot of possible values of a_1 and a_0 those satisfy $\zeta_{\min} = 0.52$ and $\omega_n = [4.75 \quad 15]$.

The values of the coefficients obtained as the solution to Equation (4.16) are,

$$
\begin{aligned}
[\underline{\alpha}_1 \quad \overline{\alpha}_1] &= [15.610 \quad 22.061] \\
[\underline{\alpha}_0 \quad \overline{\alpha}_0] &= [105.124 \quad 226.112]
\end{aligned}
\tag{4.17}
$$

The results in Equation (4.17) are plotted as the shaded area in Figure 7. The root space generated from the Edge Theorem for the coefficient values in Equation (4.17) is plotted in Figure 8. As it is shown in Figure 8, the minimum damping attained by the root spaces is about 0.52; the minimum and maximum frequencies attained are approximately 7 rad/s and 15 rad/s respectively. The domain bounded by the design parameters is the shaded area in Figure 8. Since the some of the constraints are reached, the coefficients are reasonable results.

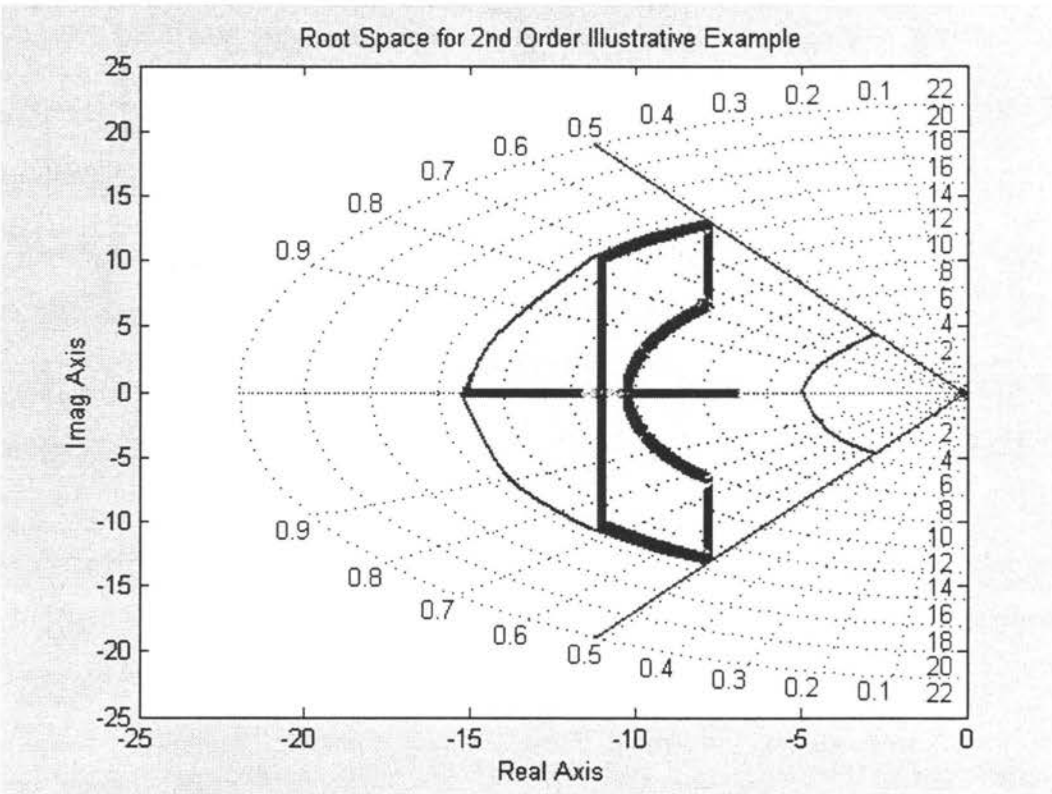

Figure 8. Plot of 2^{nd} order root space, with coefficients shown in Equation (4.20).

Step 4

The last step of the design procedure is to obtain a controller using the linear programming approach. In this example, the magnitude of the controller gain is assumed to be a design concern. So the objective function chosen for the example is to minimize the largest value in the vector K, i.e. the vector of gains in the controller. This objective function is a nonlinear function. The approach is modified to solve for the nonlinear objective function. The formulation is:

$$
\begin{aligned}
& \underset{a,b}{Min} \quad \max[K] \\
& \delta\_l \leq AK \leq \delta\_u \\
& \quad K = [K_1 \quad K_2 \quad \cdots \quad K_{2m}]^T \\
& \quad\quad = [a_1 \quad a_2 \quad \cdots \quad a_m \quad b_1 \quad b_2 \quad \cdots \quad b_m] \\
& \quad K_i \leq K_{\max} \quad ; for\, \forall\, i
\end{aligned}
\tag{4.18}
$$

Since the controller was assumed to be 0^{th} order, the controller is simply a proportional controller and is in the form of $K_{\max} = {}^{a_0}\!/_{b_0} = {}^{K_{\max}}\!/_{1}$. For this example, the linear programming problem to obtain a controller is stated Equation (4.19).

$$
\begin{aligned}
& Min \quad K_{max} \\
& subject\ to \\
& \begin{bmatrix} 15.61 \\ 105.12 \end{bmatrix} \leq \begin{bmatrix} 0.03257 & 0.9186 \\ 1.1052 & 1.064 \end{bmatrix} \begin{bmatrix} K_{\max} \\ 1 \end{bmatrix} \leq \begin{bmatrix} 22.06 \\ 226.11 \end{bmatrix}
\end{aligned}
\tag{4.19}
$$

The constraint set in Equation (4.19) is inconsistent, i.e. there is no feasible solution to the constraints. The first constraint requires $451.072 \leq K_{\max} \leq 649.107$, while the second constraint requires $95.072 \leq K_{\max} \leq 203.624$. There is no single value of $K_{\max}$ that can satisfy both inequities. Hence, the linear programming problem in Equation (4.19) is infeasible. This implies that a proportional controller (0^{th} order controller) will not achieve the design domain specified by the target interval polynomial.

Therefore, the order of the controller is raised, and the procedures in Step 2 to Step 4 are repeated. The controller order of the next design attempt is assumed to be 1^{st} order. The controller *C(s)*, the controller gain matrix *K*, and the target interval polynomial are shown in Equation (4.20), (4.21) and (4.22) respectively.

$$C(s) = \frac{a_1 s + a_0}{b_1 s + b_0} \tag{4.20}$$

$$K = [a_1 \quad a_0 \quad b_1 \quad b_0] \tag{4.21}$$

$$\delta_T(s) = s^3 + [23.72 \quad 26.81]s^2 + [230.46 \quad 240.67]s + [789.94 \quad 800.59] \tag{4.22}$$

The linear programming problem to solve for the controller is:

$$\begin{aligned} &\underset{a,b}{Min} \quad \max[K] \\ &subject\ to \\ &\begin{bmatrix} 1 \\ 789.94 \\ 230.46 \\ 23.72 \end{bmatrix} \le \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.03257 & 0 & 0.9186 & 1 \\ 1.1052 & 0.03257 & 1.064 & 0.9186 \\ 0 & 1.1052 & 0 & 1.064 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \\ b_1 \\ b_0 \end{bmatrix} \le \begin{bmatrix} 1 \\ 800.59 \\ 240.67 \\ 26.81 \end{bmatrix} \end{aligned} \tag{4.23}$$

Design Results

The controller design results are concluded in Table 4.1, and show that the settling time specification is met, while the overshoot specification is missed by 3%. The reasons of missing the overshoot specification are likely to be the round of error and the effect of the zeros. The pole-zero map, the step response and the bode plots of the resulting system are shown in Figure 9, Figure 10 and Figure 11 respectively.

Table 4.1. Summary of design results of 2nd order system example.

Order of controller (m)	1
Controller $C(s)$	$\dfrac{171.9\,s + 699.63}{s + 20.43}$
The design parameters chosen	$\zeta_{\min} = 0.52$ $\omega_n = [4.75 \quad 15]$
Resulting damping ratios	0.762, 1
Resulting frequencies	11.7, 5.81
Overshoot	18%
Settling time (seconds)	0.762
Rise time (seconds)	0.136

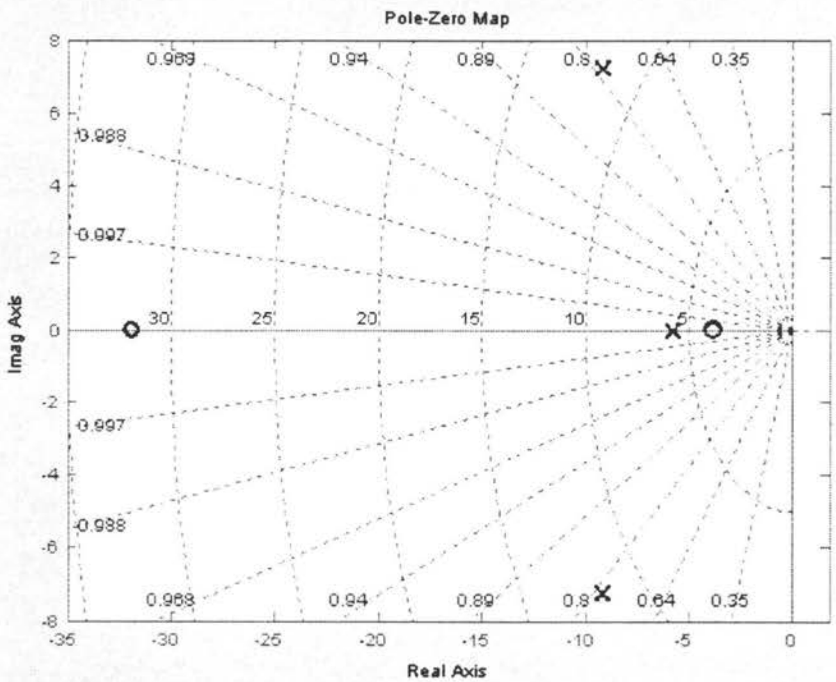

Figure 9. Pole-zero map of closed-loop system.

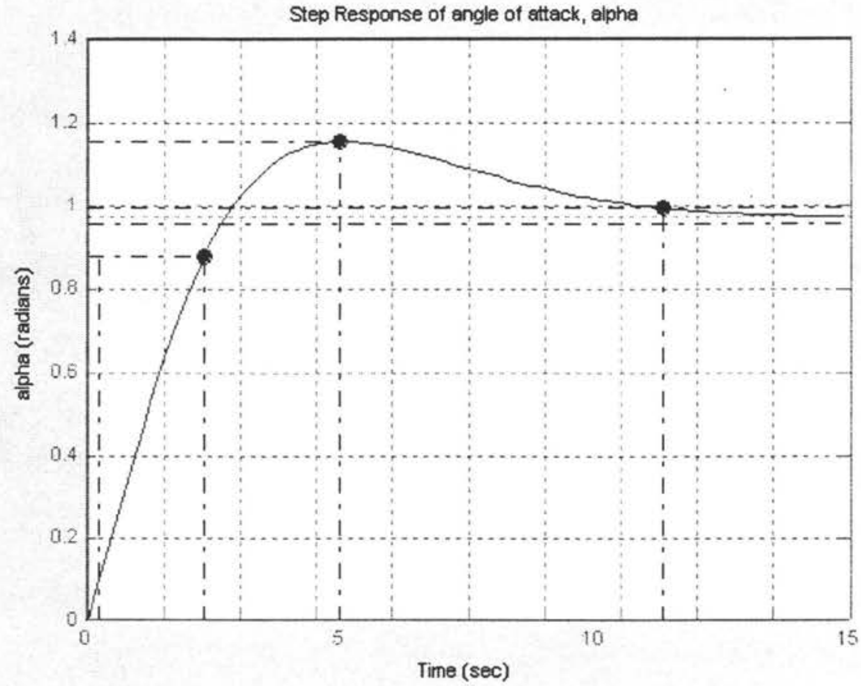

Figure 10. Step response of the closed-loop system.

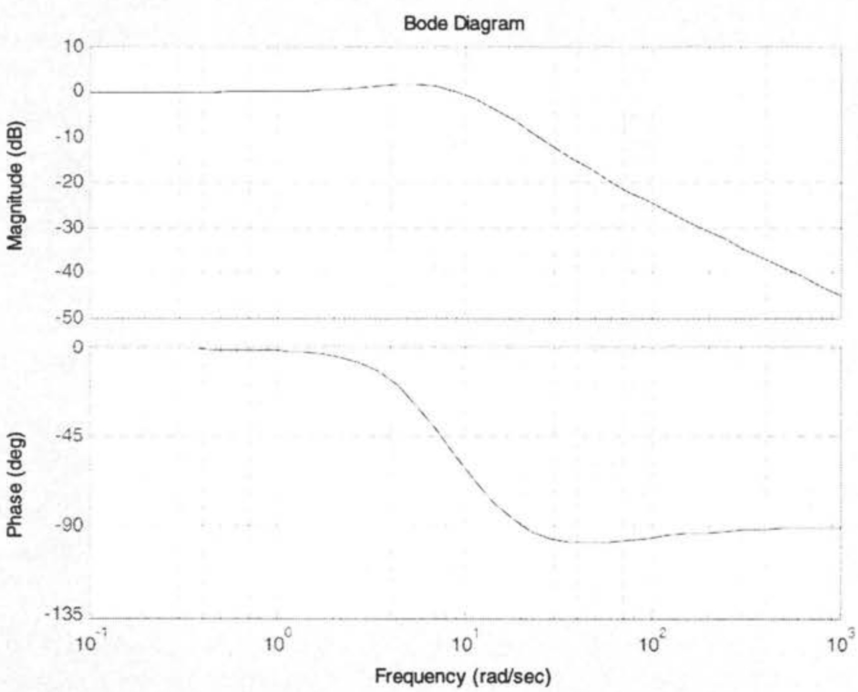

Figure 11. Bode plots of the closed-loop system

Discussions

The design results in Table 4.1 show that the overshoot requirement is not met. In order to fully satisfy the design specifications, a larger minimum damping ratio may be chosen instead. In addition, since the zeros of the system also affect the performance, the design will be more accurate if a FOMM formulation is used instead of a FOPA formulation. The algorithm can be easily transformed to a FOMM formulation by adding extra constraints on the transfer function's numerator coefficients of the resulting system if the desired zeros locations are known.

CHAPTER 5: COMPASION OF THE FIXED ORDER CONTROLLER DESIGN PROCESS (USING LP APPROACH AND INTERVAL POLYNOMAL SEARCH ALGORITHM) TO LQR

Linear quadratic regulator (LQR) is a classic optimal control design method. It obtains a controller by minimizing a chosen quadratic performance index function. In this chapter, the controller for the Boeing 747 short period mode example in Chapter 4.3 will be redesigned using LQR technique. The differences between the LQR design method and the fixed order controller design process (using the linear programming approach and the interval polynomial search algorithm) are compared.

5.1 Theories of LQR

Suppose the state space of a SISO LTI system is stated in Equation (5.1), where $A \in R^{n\times n}, B \in R^{n\times 1}$ and $C, H \in R^{1\times m}$.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ z &= Hx \end{aligned} \tag{5.1}$$

For state feedback, the regulator control law is stated in Equation (5.2), where $K \in R^{n\times m}$.

$$u = Kx \tag{5.2}$$

The performance index, J, of LQR is a quadratic weight function. Equation (5.3) shows the formula of J, where Q and R are positive semidefinite matrices. Q is the state weight matrix, while R is the input weight matrix.

$$J = \frac{1}{2}\int_0^\infty \left[x^T Q x + u^T R u\right] dt \tag{5.3}$$

The LQR technique obtains an optimal controller by minimizing the performance index given in Equation (5.3) that solves the Algebraic Riccati equation in Equation (5.4). S in Equation (5.4) is a Lagrange multiplier, and the corresponding controller, K, is defined in Equation (5.5)

$$A^T S + SA - SBR^{-1}B^T S + Q = 0 \tag{5.4}$$

$$K = R^{-1}B^T S \tag{5.5}$$

There are other variations of LQR, such as the output feedback, i.e. $u = Ky$.

5.2 Controller Design with LQR

Since LQR regulates by state feedback, the system must be controllable. To design a LQR controller, a control system designer chooses the values in the state weight matrix, Q, and the input weight matrix, R that yields a controller that gives a desired response. The Q matrix is generally associated with the system overshoot and response time, while matrix R associates with the system transient time. Multiple trials are often needed to achieve the correct response. The 2[nd] order system in Chapter 4.3 is used to illustrate an LQR design.

5.2.1 Illustrative Example (2^{nd} Order System)

The plant used and the design specifications in this example are identical to those used in the illustrative example in Chapter 3.3. The equivalent state space representation of the plant is:

$$A = \begin{bmatrix} -0.000835 & 1 \\ -0.001646 & -0.4860 \end{bmatrix}$$
$$B = \begin{bmatrix} -0.03258 \\ -1.0879 \end{bmatrix} \tag{5.6}$$

The controller is designed using the state feedback LQR technique. The design results are summarized in Table 5.1. Figure 12 shows the block diagram of the resulting system. The step response and bode plots of the resulting system are shown in Figure 13 and Figure 14 respectively.

Table 5.1. Summary of LQR design results.

Matrix Q	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Matrix R	[1]
K	$\begin{bmatrix} -0.9972 & -1.2721 \end{bmatrix}$
Overshoot	3%
Settling time (in seconds)	0.20
Rise time (in seconds)	0.0768

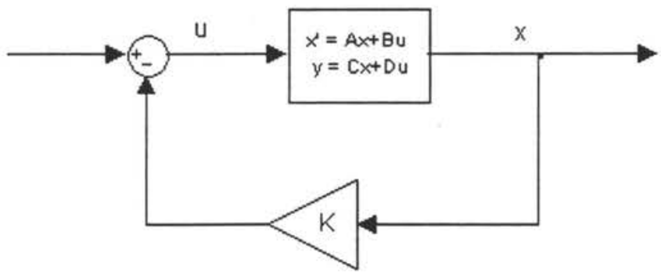

Figure 12. LQR system block diagram

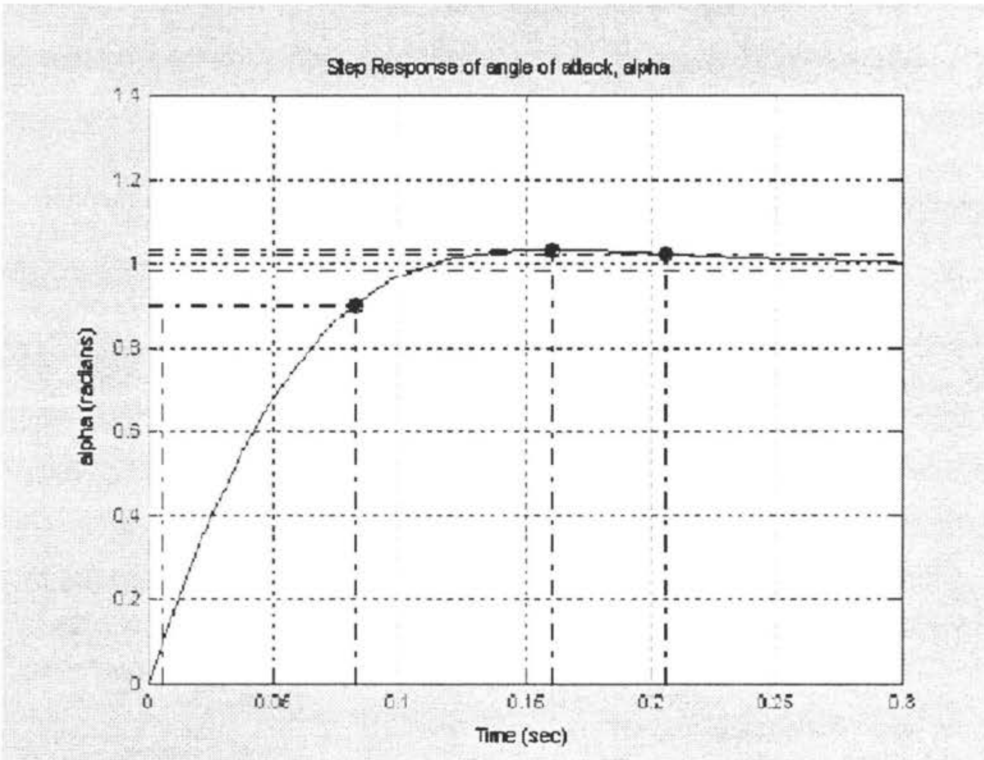

Figure 13. Step response of the system with LQR controller.

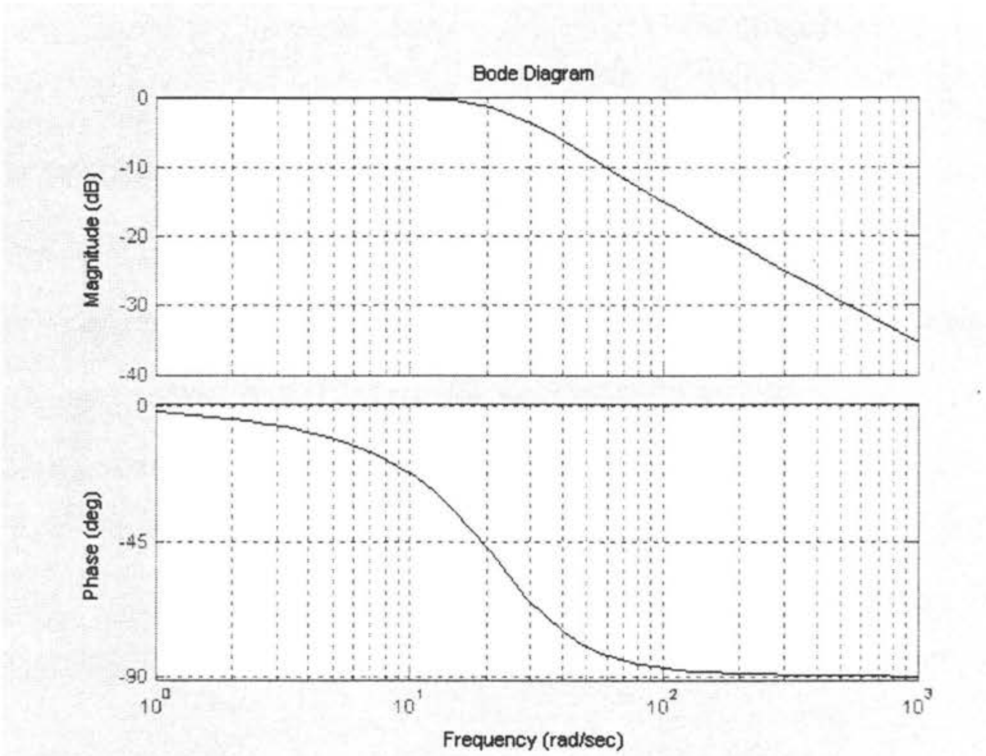

Figure 14. Bode plots of the system with LQR controller.

The controller obtained from the LQR technique yields better performances than the controller designed using the fixed order controller design process proposed in Chapter 4. The results are expected since the LQR controller uses full state feedback, i.e. feedbacks both states, while the fixed order controller feedbacks the output only.

5.3 Comparisons of LQR and Fixed Order Controller Design Process (Using LP Approach and Interval Polynomial Search Algorithm)

Both LQR and the fixed order controller design process (using the LP approach and the interval polynomial search algorithm) obtain an optimal controller by minimizing an objective function. Both methods obtain a gain feedback controller and can be applied to fixed order controller design. In addition, both of techniques can shape the time responses. The controllers designed by either method work fairly well in the illustrative example in terms of meeting the time domain design specifications.

The LQR technique is a quadratic optimal canonical problem. It obtains a controller by minimizing an integral of weighted states and inputs function. The design parameters are Matrix Q and Matrix R, which determine the emphasis on certain states or inputs. Therefore, design aspects, such as the control power and effort of a specific state or input, can be addressed conveniently. Different controller and performances are yielded by different Q and R values. The time domain specification is implied in the LQR problem formulation, through the choice of Q and R. However, the correlation of time response yielded by the design and the weight matrices Q and R is implicit. Control engineers often have to perform several iterations on Q and R in order to achieve the desired response.

In comparison, the fixed order controller design process proposed in Chapter 4 incorporates the linear programming approach and the interval polynomial search algorithm (in Chapter 3). Its formulation is based on the fixed order pole assignment (FOPA) problem and the fixed order model matching (FOMM) problem. The time domain specifications are identified in the target interval characteristic polynomial obtained by the interval polynomial search algorithm. Control engineer only has to pick a range of natural frequencies and a minimum damping ratio. The relationship of frequencies, damping ratios and time response is more intuitive. Therefore, from the view of fulfilling the time response requirements, the design process developed in Chapter 4 may be more insightful than the LQR techniques. Furthermore, it maintains the most important advantage of the linear programming approach design method: not only are the design specifications fulfilled, the controller design process has a lot of freedom in specifying various design objectives. However, the described controller design process relies on the interval polynomial searched based on the approximated correlations of frequencies, damping ratio and time response specifications. The approximation becomes less accurate as the order of the system increased. The performance of the approach may decline as the order of the system rises.

To conclude the discussion the above discussion, LQR is a well studied optimal control technique. It minimizes an integral of weighted states and inputs function and results in a controller. The linear programming approach to controller design is a relatively new approach. The controller design process proposed in the previous chapter focuses only on fixed order controller design. Its objective function is user-defined. However, the set of constraints was formulated to solve for a fixed order controller. There are researches on linear programming approach to robust controller which minimizes the sensitivity of a system. All of these described techniques are subsets of the linear programming controller design approach.

CHAPTER 6: APPLYING FIXED ORDER CONTROLLER DESIGN PROCESS (USING LP APPROACH AND INTEVAL POLYNOMIAL SEARCH ALGORITHM) TO HIGHER ORDER SYSTEM

The revised linear programming design approach is applied to a 4th order system to verify if it is applicable to higher order system. The system chosen is a Boeing 747 jet transporter. The data is taken from Etkin[5]. Considering only the altitude control, the transfer function of the vertical velocity to the elevator deflection is:

$$P(s)=\frac{w}{\delta_e}=\frac{17.85s^3+904.0s^2+6.208s+3.445}{s^4+0.7505s^3+0.9355s^2+0.009463s+0.004196}\,\frac{ft/s}{rad} \tag{6.1}$$

Open Loop Plant Analysis

The step response and bode plots of the plant shown in Equation (6.1) is illustrated in Figure 15. The open loop plant performances are summarized in Table 6.1.

Table 6.1. Open loop 4th order system performance summary.

Short period mode damping	0.387
Phugoid mode damping	0.0489
Short period mode frequency	0.962
Phugoid mode frequency	0.0673
Overshoot	50.6%
Rise time(in seconds)	1.27

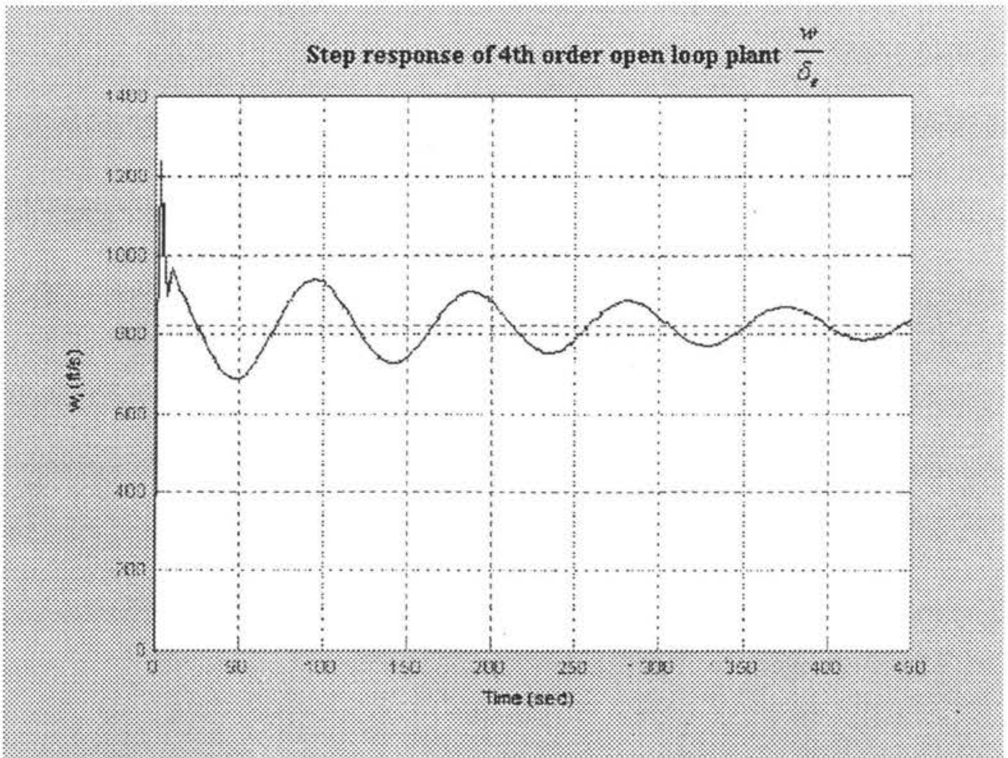

Figure 15. Step response of the open loop 4^{th} order plant.

The open loop Boeing 747 4^{th} order system consists of both phugoid and short period mode. The rise time is considered to be reasonable. However, there is a 50% overshoot in the system. For a jet transporter, large overshoot is a concern since it causes discomfort to passengers. A controller is needed to suppress the system overshoot. Therefore, the design specification for this example is:

- Overshoot, $M_p \leq 0.25$

Design Results

The design procedures were described in Chapter 4.2. The initial order of the controller was 0, and it was increased until the linear programming problem in Equation (4.4) was feasible. The objective function chosen for the linear programming problem was to minimize the sum of all the controller gains. The design parameters chosen were: $\zeta_{\min} = 0.4, \omega_n = [0.5 \quad 25]$. The results of the design are summarized in Table 6.2. The step response of the resulting system is shown in Figure 16.

The overshoot of the system was suppressed to less than 20%. Hence, the design specification was attained. The settling time of the response was retained. For a shorter settling time, the natural frequency range, ω_n, chosen for design should be decreased.

Table 6.2. Summary of design results for 4th order plant.

Order of controller (*m*)	3
Controller *C(s)*	$\dfrac{2.118e05s^3 + 2.789e07s^2 + 2.082e07s + 2.57e07}{s^3 + 3.78e06s^2 + 6.864e08s + 2.507e10}$
The design parameters chosen	$\zeta_{\min} = 0.4$ $\omega_n = [0.5 \quad 25]$
Target characteristic coefficients, lower bound values	[64 1982 35918 396601 2477964 8825760 15984000]
Target characteristic coefficients, upper bound values	[69 2135 37611 396748 2644054 9909420 16309800]
Overshoot	19%
Rise time (seconds)	0.896

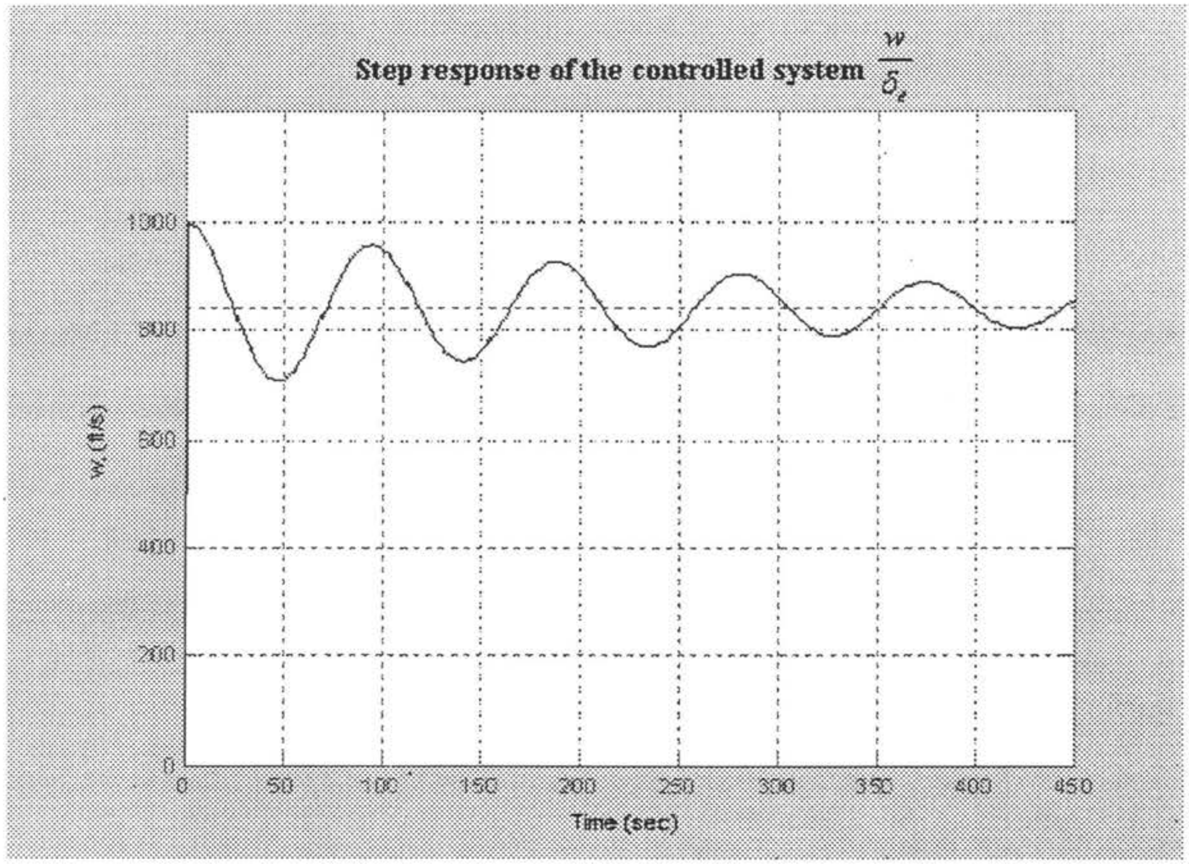

Figure 16. Step response of the closed-loop 4th order plant.

The root space of the targeted interval characteristic polynomial is plotted in Figure 17. The domain bounded by the chosen parameters, i.e. minimum damping ratio, minimum and maximum frequencies, is shaded.

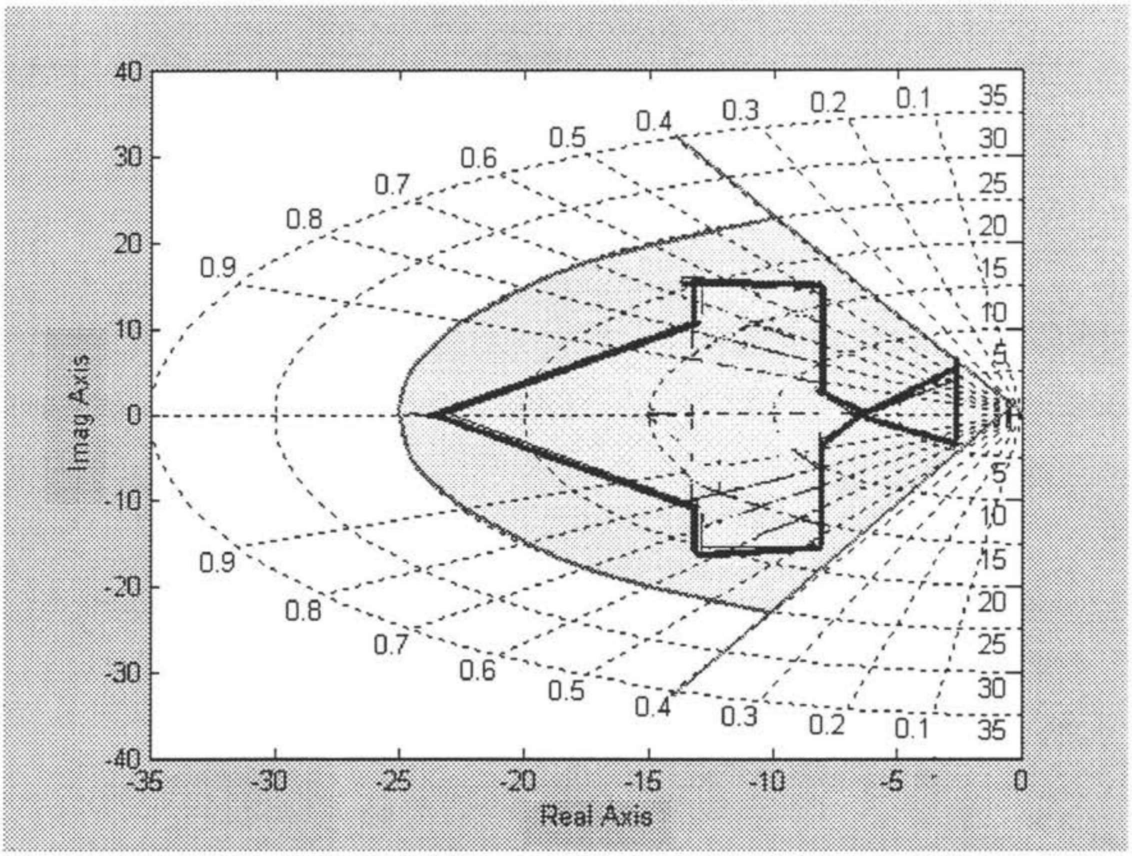

Figure 17. Root space of the target interval characteristic polynomial in Table 6.2.

Discussions

The illustrative examples in this chapter as well as Chapter 4.3 show that the fixed order controller design algorithm approach works for 2[nd] order and 4[th] order plants. Therefore, it can be concluded that the proposed algorithm on fixed order controller design is applicable to a plant of any order.

The yielded controller is substantially dependent on the interval polynomial resulting from maximizing the root space within the chosen design parameters. Therefore, a lower order controller is possible to be obtained if the design parameters are relaxed. However, there is a trade off in system performances.

CHAPTER 7: CONCLUSIONS

A fixed order controller design process was proposed. The linear programming approach to fixed order controller design based on the fixed order pole assignment problem (FOPA) and fixed order model matching problem (FOMM). The proposed controller design process closely correlates the physical time domain performances, such as overshoot and settling time to the controller design methodology. The design parameters to be selected for the proposed controller design process are minimum damping ratio and range of natural frequency. These parameters have explicit correlations with time domain specifications. Hence, the selection of the parameters is insightful.

The interval polynomial search algorithm developed in Chapter 3 suggested a scheme to incorporate the damping ratio and natural frequencies specifications into the linear programming approach to fixed order controller design. The constraint values in the linear programming approach are the coefficients of the target characteristic interval polynomial found by the interval polynomial search algorithm. The scheme of the interval polynomial search algorithm's formulation was to maximize the range of each coefficient using a constrained nonlinear programming method. The constraints of the nonlinear programming problem were formulated based on the Edge Theorem. It restricted the roots of all the vertex polynomials to stay within the design domain, where the domain was defined by a specified minimum damping ratio and a range of allowable natural frequencies.

A 2^{nd} order system was used to demonstrate the complete design process. The developed controller design process was shown to be applicable to higher order systems by applying it to a 4^{th} order plant. The controller design process using the LP approach and interval search polynomial algorithm was compared with LQR. The design parameters of the LQR method are matrices Q and R. These matrices represent the desired emphasis on states and inputs. However, the correlations of time response to matrices Q and R are ambiguous. Several iterations on Q and R are often needed to achieve the time domain performance specifications. On the other hand, the design parameters for design process proposed in this thesis are damping ratio and natural frequency. These parameters have a closer connection with time domain performance specifications.

To conclude, the fixed order controller design process proposed in this thesis uses the linear programming approach to fixed order controller design developed by Keel and Bhattacharyya [1, 2] and the interval polynomial search algorithm described in Chapter 3. The interval polynomial search algorithm in Chapter 3 is the bridge that closes the gap between the mathematical design approach and time domain performance specifications. The design parameters of the design process to be selected by control engineers are insightful. The controller design process is also capable in searching for the lowest order controller for any specific design domain. A lower order controller can be yielded if the design parameters are relaxed. The proposed controller design process maintains the benefits of the linear programming design approach by Keel and Bhattacharyya. Its most important advantage is the great freedom in addressing a single or multiple design emphasis.

CHAPTER 8: FUTURE WORKS

The formulation of the controller design process proposed in this thesis was based on the FOPA with a SISO LTI system. It can be extended to solve the FOMM problem as well as for MIMO systems. In order to extend the algorithm to solve for FOMM problem, the zero locations have to be specified and included in the constraints of the linear programming problem. The procedures were discussed in Chapter 2 and 4. However, the extension is not simple, as the relation between time performance and zeros locations is uncertain. For a MIMO system, there are multiple resulting transfer functions. Since all of these transfer functions have a common characteristic equation, the fixed order controller design algorithm can be used to solve for a MIMO system as well. However, the approximated correlation of damping ratio, frequency and time response specifications becomes less accurate as the complexity of the system increases, the performance of the algorithm become less desirable. The resulting controller may not achieve all the design specifications. Further studies on extending the algorithm to FOMM problems and MIMO systems are needed.

Keel and Bhattacharyya had pointed out that the linear programming controller design approach guarantees robust stability and performance [1, 2, 7, 8]. They had also formulated the LP approach to deal with the uncertainties in their works [1, 2]. Further analysis on the robust stability and performance of the controllers obtained from the controller design process proposed is this thesis is needed.

APPENDIX A: DATA FOR BOEING 747-100

These data are taken from Etkin [5].

Table A.1. Boeing 747-100 data.

Planform area, S (ft^2)	5500
Wing span, b (ft)	195.68
Chord length, $\bar{c}$ (ft)	27.31
h	0.25
Altitude (ft)	20000
M	0.5
Velocity, V (ft/s)	518
Weight, W (lb)	6.366 e05
I_x (slug ft^2)	1.82 e07
I_y (slug ft^2)	3.31 e07
I_z (slug ft^2)	4.97 e07
I_{zx} (slug ft^2)	9.70 e05
ξ (degrees)	-6.8
C_D	0.040

Table A.2. Boeing 747-100 longitudinal dimensional derivatives.

	X (lb)	Z (lb)	M (ft lb)
u (ft/s)	-4.883 e01	-1.342 e03	8.176 e03
w (ft/s)	1.546 e03	-8.561 e03	-5.627 e04
q (rad/s)	0	-1.263 e05	-1.394 e07
$\dot{w}$ (ft/ s^2)	0	3.104 e02	-4.138 e03

APPENDIX B: MATLAB CODES FOR PLOTTING ROOT SPACE OF 2ND ORDER INTERVAL POLYNOMIAL

```
% second order polynomial; f(s)=s^2+a1*s+a0
%a= [min_ao, max_ao, min_a1, max_a1]

%Edge theory
lamda=0;
i=1;
while (lamda<=1),
   v12=[1 a(3) a(1)]*lamda+[1 a(3) a(2)]*(1-lamda);
   v13=[1 a(3) a(1)]*lamda+[1 a(4) a(1)]*(1-lamda);
   v14=[1 a(3) a(1)]*lamda+[1 a(4) a(2)]*(1-lamda);
   v23=[1 a(3) a(2)]*lamda+[1 a(4) a(1)]*(1-lamda);
   v24=[1 a(3) a(2)]*lamda+[1 a(4) a(2)]*(1-lamda);
   v34=[1 a(4) a(1)]*lamda+[1 a(4) a(2)]*(1-lamda);

   R12(:,i)=roots(v12);
   R13(:,i)=roots(v13);
   R14(:,i)=roots(v14);
   R23(:,i)=roots(v23);
   R24(:,i)=roots(v24);
   R34(:,i)=roots(v34);

   lamda=lamda+0.001;
   i=i+1;
end
figure
plot(real(R12),imag(R12),'bx')
hold on
plot(real(R13),imag(R13),'gx')
hold on
plot(real(R14),imag(R14),'rx')
hold on
plot(real(R23),imag(R23),'cx')
hold on
plot(real(R24),imag(R24),'mx')
hold on
plot(real(R34),imag(R34),'yx')
hold on
sgrid
```

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